Technical Annex Statistical Assessment Samples (SAS)
of the Concept Additional Tests (CAT):

Procedure "Statistical Assessment of Sample Test Results"

SAS 1 Introduction

This document describes a procedure for the statistical assessment of burst and load cycle tests intended for evaluation within the "concept additional tests" (CAT). The main purpose is to consider mean and scatter values as well uncertainties due to a limited sample size for safety assessments.

The described statistical procedure is based on sample test results from Slow Burst Tests (SBT) according to CAT 5 and Load Cycle Test (LCT) according to CAT 6. The procedure includes the calculation of mean and scatter values. These sample properties will be compared to a dedicated minimum survival rate SR_{min} according to CAT 2. The defined minimum survival rate is valid for the whole service life until end of life (EoL). Composite pressure receptacles at beginning or mid of life have (BoL or MoL) to provide a proper reserve due to degradation of their strength properties over lifetime.

A safety assessment with limited sample size generally includes an uncertainty regarding the true values of the tested design type. This uncertainty strongly depends on the number of tested specimens and can be reduced by an increase of sample size. The following procedure considers the sample size as part of the safety assessment by implementation of a defined confidence level.

Recommended sample size for a safety assessment is 5 according to CAT 4. The procedure covers the assessment of samples with a sample size different from 5 as well. A higher sample size reduces uncertainty and therefore leads to a reduction of required sample properties for exceeding the minimum survival rate.

SAS 2 Scope

The procedure is suitable for all types and sizes of composite pressure receptacles produced in large numbers. In general this is assumed for design types not exceeding a water capacity of 450 litres.

Design types which are not assessable by sufficient sample testing due to their limited production numbers shall be examined and monitored with an equivalent procedure in accordance with the competent authority.

SAS 3 Statistical Assessment of SBT and LCT

The following chapters describe a method to quantify the survival rate of samples tested in accordance with CAT 5 (Slow Burst Test) and/or CAT 6 (Load Cycle Test). The method is based on the calculation of sample mean and scatter properties (SAS 3.1) and the comparison by graphical or analytical assessment as described in SAS 3.2. The resulting properties of a sample are intended to be compared with the minimum survival rate SR_{min} (defined in CAT 2) for the relevant test.

Note: The statistical description and assessment of test results are elaborated employing examples in [1] to [6].

SAS 3.1 Calculation of statistical sample properties

In the following, the sample size means the number of specimen per sample and is named n. The test result $i = 1, 2, 3 \ldots n$ for each specimen is given. The sample shall be evaluated for their mean and scatter value as defined in table **SAS-1**. It is recommended to perform an outlier test according to **SAS 3.4** prior to the assessment. Extreme outliers shall be assessed concerning their relevance for safety of the relevant population of pressure receptacles (compare SAS 3.4). Conclusions shall be discussed with the competent authority.

Table SAS-1: Calculation of sample parameters for SBT and LCT

Calculation of sample parameters	
Slow Burst Test	**Load Cycle Test**
List of symbols	
MSP = Maximum Service Pressure MSP is dependent on the design type approval: Either MSP means the test pressure PH in case of an approval for general service or MSP means the developed gas pressure at maximum allowable settled temperature (65 °C) in case of a design type approval for dedicated gas service.	
n = sample size p_i = burst pressure of an individual specimen (Result of one burst test) m_{pB} = Mean value of burst pressure s_{pB} = Standard deviation of burst pres. $\Omega_{50\%}$ = Relative mean of burst pressure Ω_s = Relative scatter of burst pressure	n = sample size N_i = Number of load cycles to leakage of individual specimen (Result of a hydraulic load cycle test to PH or MSP) m_N = mean value of log- LCT s_N = Standard deviation of log-LCT $N_{50\%}$ = Median of load cycle results N_s = Scatter of load cycle results
Calculation of sample mean	
$m_{pB} = \frac{1}{n}\sum_{i=1}^{n} p_i$ $\Omega_{50\%} = \frac{m_{pB}}{MSP}$	$m_{\log N} = \frac{1}{n}\sum_{i=1}^{n}\log_{10}(N_i)$ $N_{50\%} = 10^{m_{\log N}}$
Calculation of sample scatter	
$s_{pB} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(p_i - m_{pB}\right)^2}$ $\Omega_s = \frac{s_{pB}}{MSP}$	$s_{\log N} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\log_{10}(N_i) - m_{\log N}\right)^2}$ $N_s = 10^{s_{\log N}}$

SAS 3.2 Assessment of survival rate

After having quantified the sample properties in accordance with SAS 3.1, they can be drawn into the relevant performance chart: Either in **Fig. SAS-1** and **Fig. SAS-2** for burst test results (SBT) or in **Fig. SAS-3** and **Fig. SAS-4** for cycle test results (LCT). There each sample is represented by one dot, independent from the sample size.

Note: These performance charts are supplemented by lines of constant survival rates SR. These lines permit an estimation of the SR of the whole population within the uncertainty expressed by the confidence level. The assessment of confidence depends on the sample size.

The survival rate SR_{min} as demanded in CAT 2 is represented by a curve. SR_{min} need to be separately determined for SBT and LCT according to CAT 2. The safety of a population deems to meet requirements as requested in CAT if the sample dot is found to be above the curve, relevant for SR, sample size and confidence level.

The performance charts Fig. SAS-1 (SBT) and Fig. SAS-3 (LCT) already considers the uncertainty due to a limited sample size of 5 specimens. If the sample dot is found above the relevant curve for SR_{min}, the survival rate of the tested design type is higher than SR_{min} with a probability of at least 95%.

Note: A higher sample size usually reduces the values to be demonstrated for mean and scatter, due to a reduction of statistical uncertainty.

In case of a sample size different from n = 5 the sample should be checked with an adjusted performance chart (e.g. Fig. SAS-2 or Fig. SAS-4) or has to fulfil the requirement as shown in **table SAS-2**. Values for k_{BT} and k_{LC} for different sample size n can be found in **table SAS-3** and **table SAS-4**.

Note: Minimum survival rates SR_{min} as defined in CAT and deduced performance charts and tables as presented here focuses on end of life (EoL). Cylinders at begin or mid of life have to provide a proper reserve to cover expected degradation during service.

Table SAS-2: *Check of sufficient survival rate*

Check of sample survival rate in compliance with requirements in CAT and a confidence level of 95%	
Slow Burst Test	**Load Cycle Test**
Requirement $SR_{SBT} > SR_{\min}$ is met if $\Omega_{50\%} - k_{BT} \cdot \Omega_s > 1$	Requirement $SR_{LCT} > SR_{\min}$ is met if $m_{\log N} - k_{LC} \cdot s_{\log N} > 0$

Table SAS-3: *values for k_{BT} for different sample sizes*

k_{BT} for confidence level 95%			
n	SR_{min} = 1 - 10^{-6}	1 - 10^{-7}	1 - 10^{-8}
2	79.6	86.7	93.2
3	22.6	24.6	26.4
4	15.1	16.4	17.6
5	12.2	13.3	14.3
6	10.8	11.7	12.5
7	9.8	10.7	11.5
10	8.4	9.1	9.8
15	7.4	8.0	8.6
20	6.9	7.5	8.1
∞	4.75	5.2	5.61

Table SAS-4: *values for k_{LC} for different sample size*

k_{LC} for confidence level 95%					
n	SR_{min} = 1-10^{-4}	1-10^{-5}	1-10^{-6}	1-10^{-7}	1-10^{-8}
2	119.9	150.7	181.5	212.2	243.0
3	33.9	42.4	51.0	59.6	68.2
4	22.5	28.2	33.9	39.5	45.2
5	18.3	22.9	27.5	32.1	36.7
6	16.1	20.1	24.2	28.3	32.4
7	14.7	18.5	22.2	25.9	29.6
10	12.6	15.8	19.0	22.2	25.4
15	11.1	14.0	16.8	19.6	22.5
20	10.4	13.1	15.7	18.4	21.1
∞	7.3	9.25	11.2	13.1	15.1

SAS 3.3 Comparison of two samples by parameter test

The parameter test as defined in **table SAS-5** permits a rough comparison of mean and scatter values of two samples. The statistical significance of this test is around 95%. That means in case of a successful test, the true values of the tested pressure receptacles are with a probability of at least 95% different from each other.

The sample size for a comparison of mean values should be at least 5 specimens for each sample. For a sufficient comparison of scatter values, a sample size of at least 10 specimens for each sample is recommended.

The parameter test can be used for the comparison of e. g. two production batch samples according to CAT 10. In this case, sample 1 contains the results out of the initial batch and sample 2 is a follow up sample representing e .g. the annual production batch test results.

Table SAS-5: *Similarity Test*

Similarity test of two samples			
Slow Burst Test		**Load Cycle Test**	
Mean value of sample 2 is significantly lower than sample 1 if:			
$\Omega_{50\%,1} > \Omega_{50\%,2} + T_m \sqrt{\frac{{\Omega_{s1}}^2}{n_1} + \frac{{\Omega_{s2}}^2}{n_2}}$		$m_{logN,1} > m_{logN,2} + T_m \sqrt{\frac{{s_{logN,1}}^2}{n_1} + \frac{{s_{logN,2}}^2}{n_2}}$	
with:	$T_m = 1.8$ $T_m = 1.7$ $T_m = 1.6$	if $n_1 + n_2 \leq 10$ if $n_1 + n_2 > 10$ if $n_1 + n_2 \geq 25$	
Scatter of sample 2 is significantly higher than sample 1 if:			
$T_s < \Omega_{s2} / \Omega_{s1}$		$T_s < s_{N2} / s_{N1}$	
with:	$T_s = 2.5$ $T_s = 2.0$ $T_s = 1.8$ $T_s = 1.5$	if $n_1 + n_2 \leq 10$ if $n_1 + n_2 > 10$ if $n_1 + n_2 \geq 15$ if $n_1 + n_2 \geq 20$	

SAS 3.4 Outlier test for extreme values

The following test procedure in accordance with GRUBBS (**table SAS-6**) is able to detect single outliers within a sample of burst or load cycle test results. The statistical significance of this test is around 95%. That means in case of a successful test the detected value is an out-liner with a probability of at least 95%.
The application of the test requires to search the maximum or minimum value of the sample and to calculate the absolute differences between this value and the sample mean. The re-sult is divided by the standard deviation of the sample. If the result is higher than the critical value G, the corresponding value can be considered an outlier. The critical values are shown in the following **Fig. SAS-0**.
In case of suspicion of significant deviations which cannot be detected by this method a graphical assessment of sample results should be applied (e.g. run chart, box plot, normal probability plot or histogram).

Table SAS-6: *Outlier test according to GRUBBS*

Outliner Test for maximum and minimum values	
Slow Burst Test	**Load Cycle Test**
Minimum burst pressure is an outlier if: $\frac{m_{pB} - min(p_i)}{s_{pB}} > G$	Minimum load cycle result is an outlier if: $\frac{m_{logN} - log_{10}(min(N_i))}{s_{logN}} > G$
Maximum burst pressure is an outlier if: $\frac{max(p_i) - m_{pB}}{s_{pB}} > G$	Maximum load cycle result is an outlier if: $\frac{log_{10}(max(N_i)) - m_{logN}}{s_{logN}} > G$

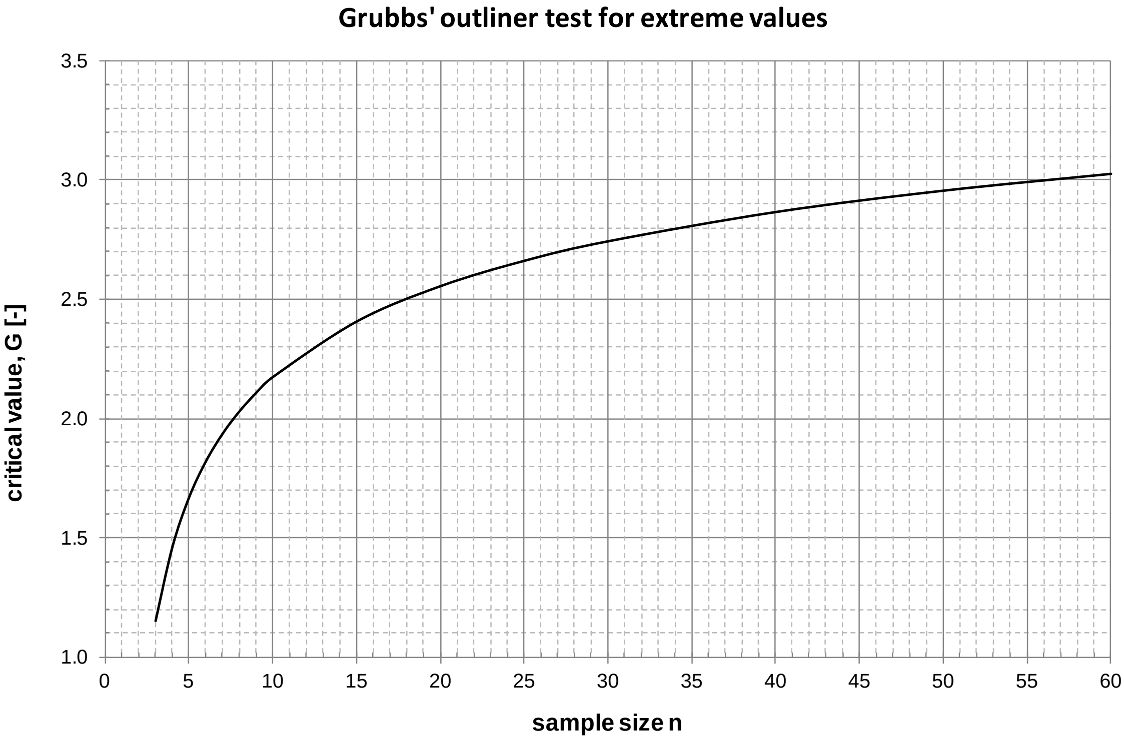

Fig. SAS-0: Critical values for Grubbs' outlier test for a statistical significance of 95%

Literature

[1] Mair, G. W.; Hoffmann, M.; Schönfelder, T.:
The Slow Burst Test as a Method for Probabilistic Quantification of Cylinder Degradation
proceedings of the „International Conference on Hydrogen Safety ICHS 2013“, Brussels: Conference Proceedings 2013; ID 102; http://www.ichs2013.com/

[2] Mair, G. W., Hoffmann, M., Scherer, F., Schoppa, A., Szczepaniak, M.:
SLOW BURST TESTING OF SAMPLES AS A METHOD FOR QUANTIFICATION OF COMPOSITE CYLINDER DEGRADATION
Elsevier: International Journal of Hydrogen Energy;
http://dx.doi.org/10.1016/j.ijhydene.2014.04.016 and Vol. 39 (2014), Issue NN, p. NN
ISSN 0360-3199

[3] Mair, G. W.; Hoffmann, M.:
Statistic Evaluation of Sample Test Results to Determine Residual Strength of Composite Gas Cylinders;
MP Materials Testing, Volume 55 (2013) No 10, p. 728 - 736; ISSN 0025-5300

[4] Becker, B.; Mair G. W.:
Einfluss der Stichprobengröße auf die Bewertung der Überlebenswahrscheinlichkeit von Druckgefäßen aus Verbundwerkstoffen;
Technische Sicherheit (TS) Bd. 4 (2014), Nr. 7/8 (Juli/Aug.), S. NN - NN.

[5] Mair, G. W.; Scherer, F.:
Statistische Bewertung von Prüfergebnissen zur Restfestigkeitsbetrachtung von Composite-Druckgefäßen
Technische Sicherheit (TS) Bd. 3 (2013), Nr. 7/8 (Juli/Aug.), S. 41 - 49.
ISSN 2191-0073

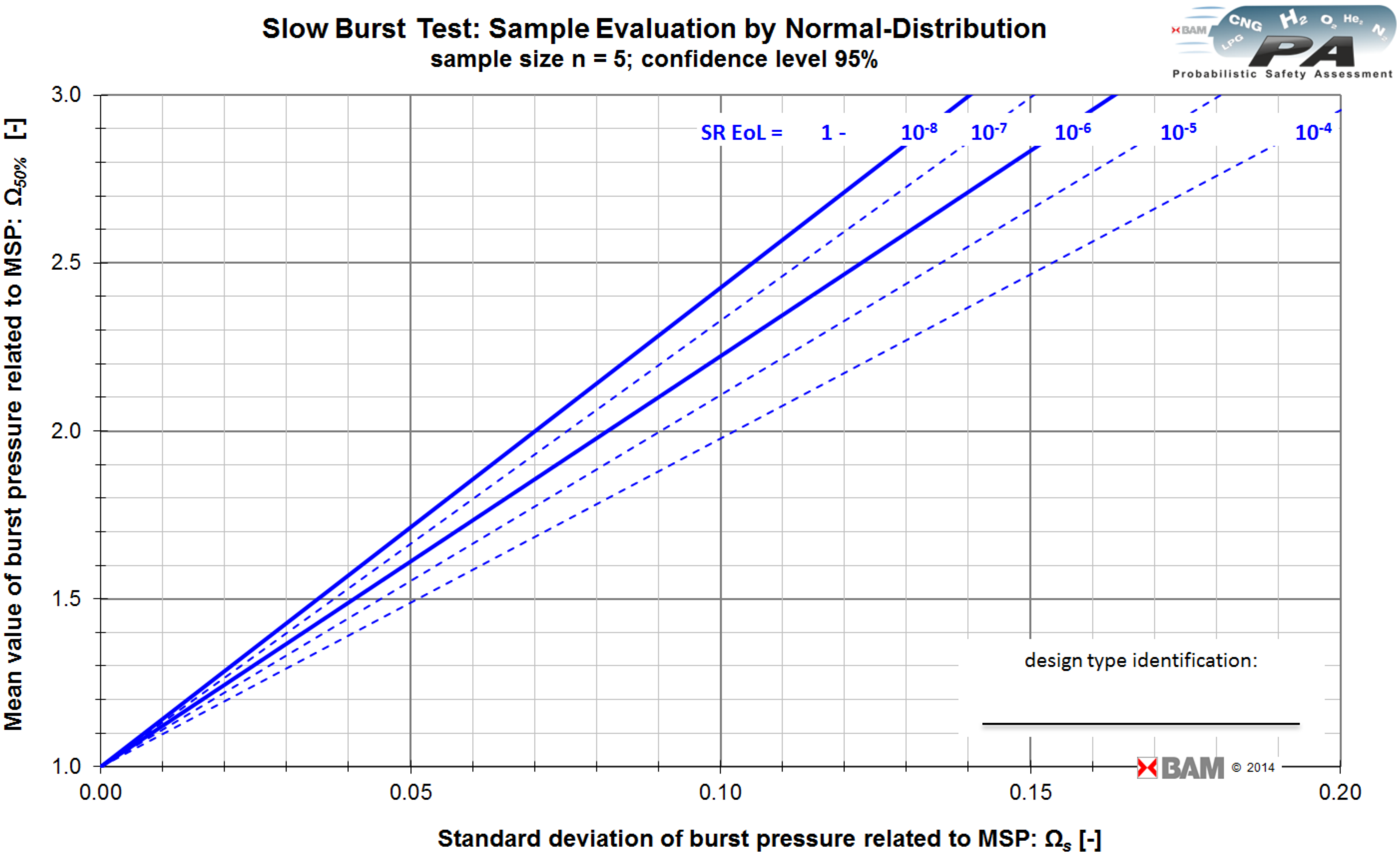

Fig. SAS-1: Performance chart for a graphical assessment of results from sample testing by SBT, sample size n = 5

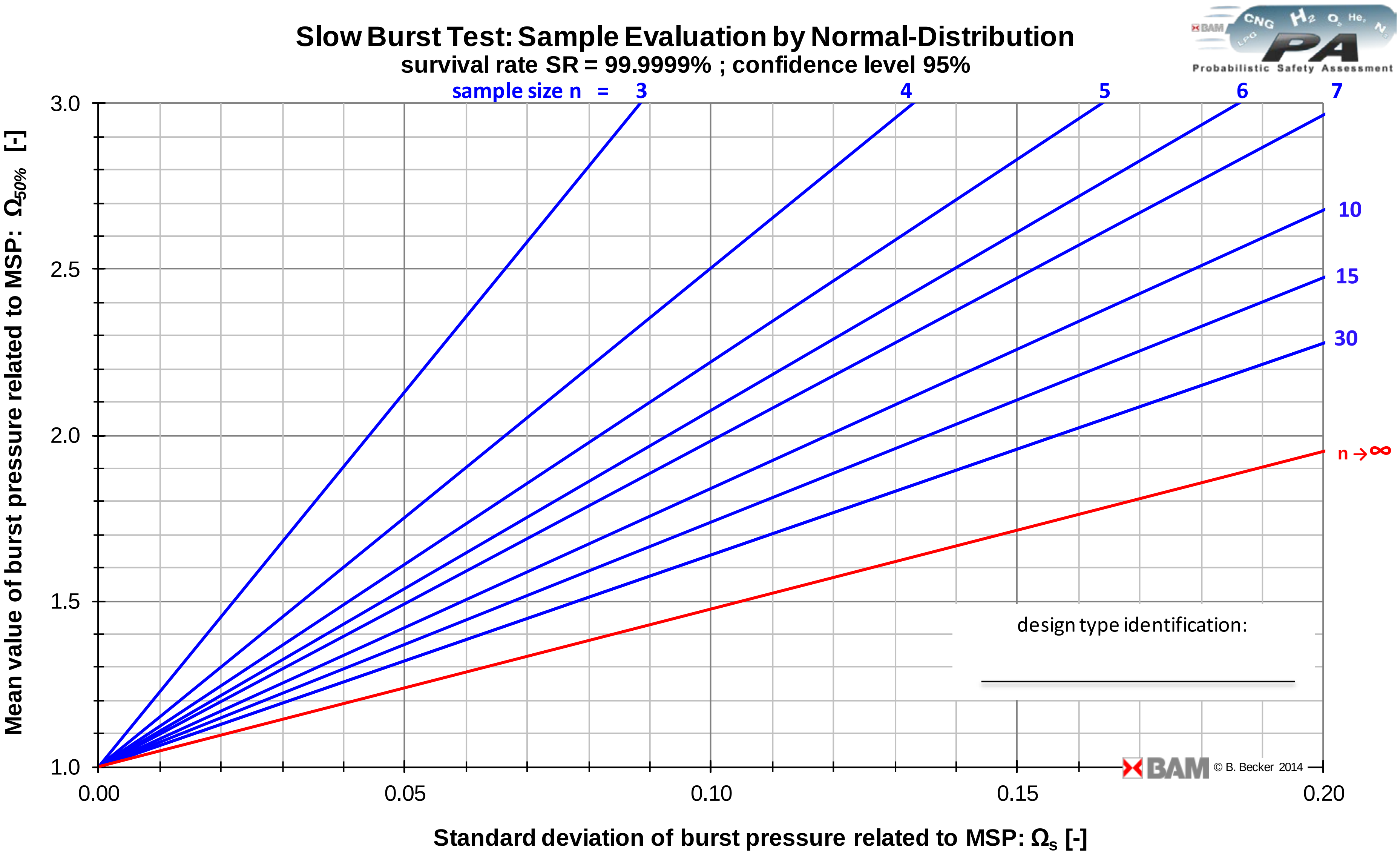

Fig. SAS-2: Performance chart for a graphical assessment of results from sample testing by SBT, survival rate SR = 99.9999%

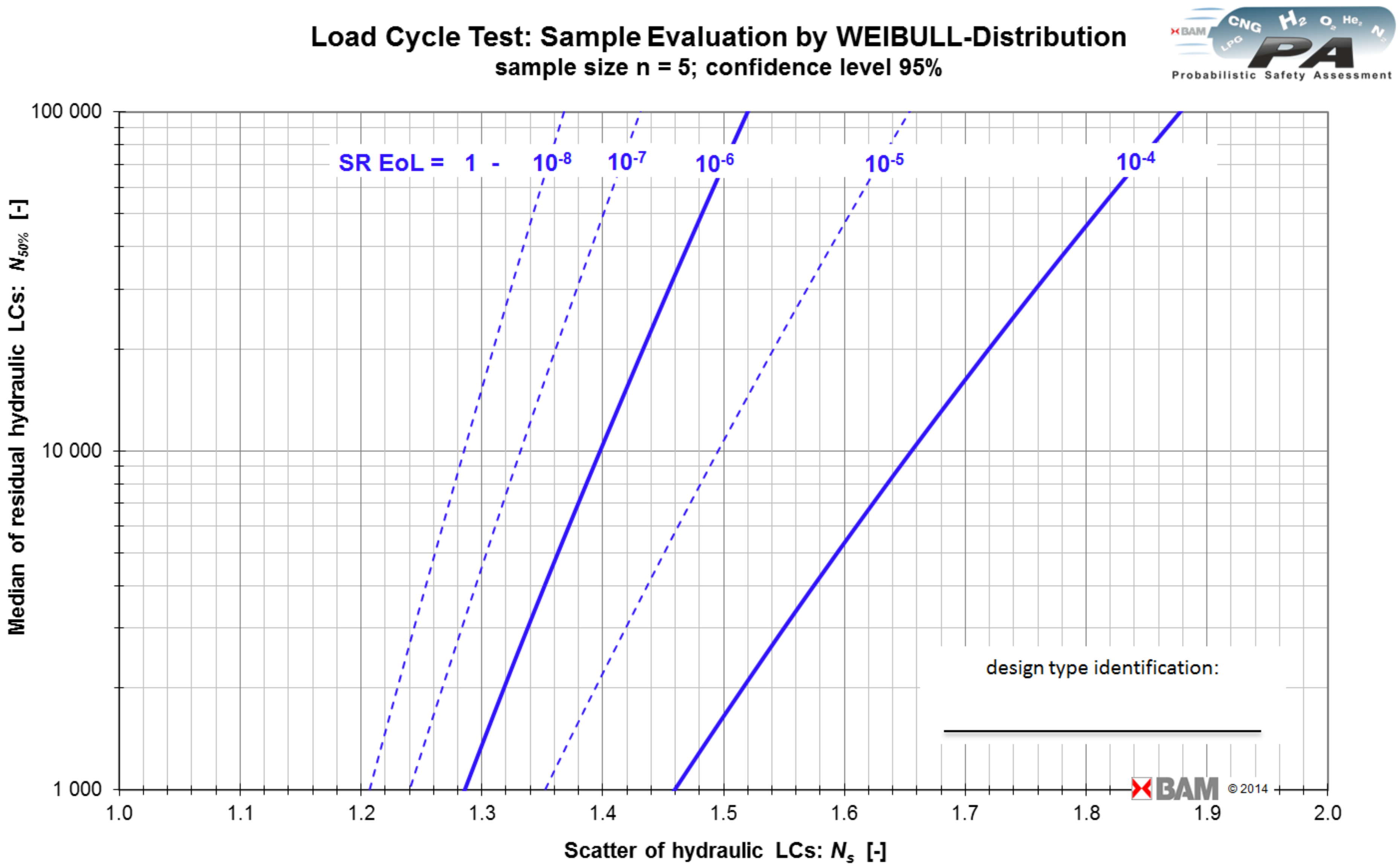

Fig. SAS-3: Performance chart for a graphical assessment of results from sample testing by LCT, sample size n = 5

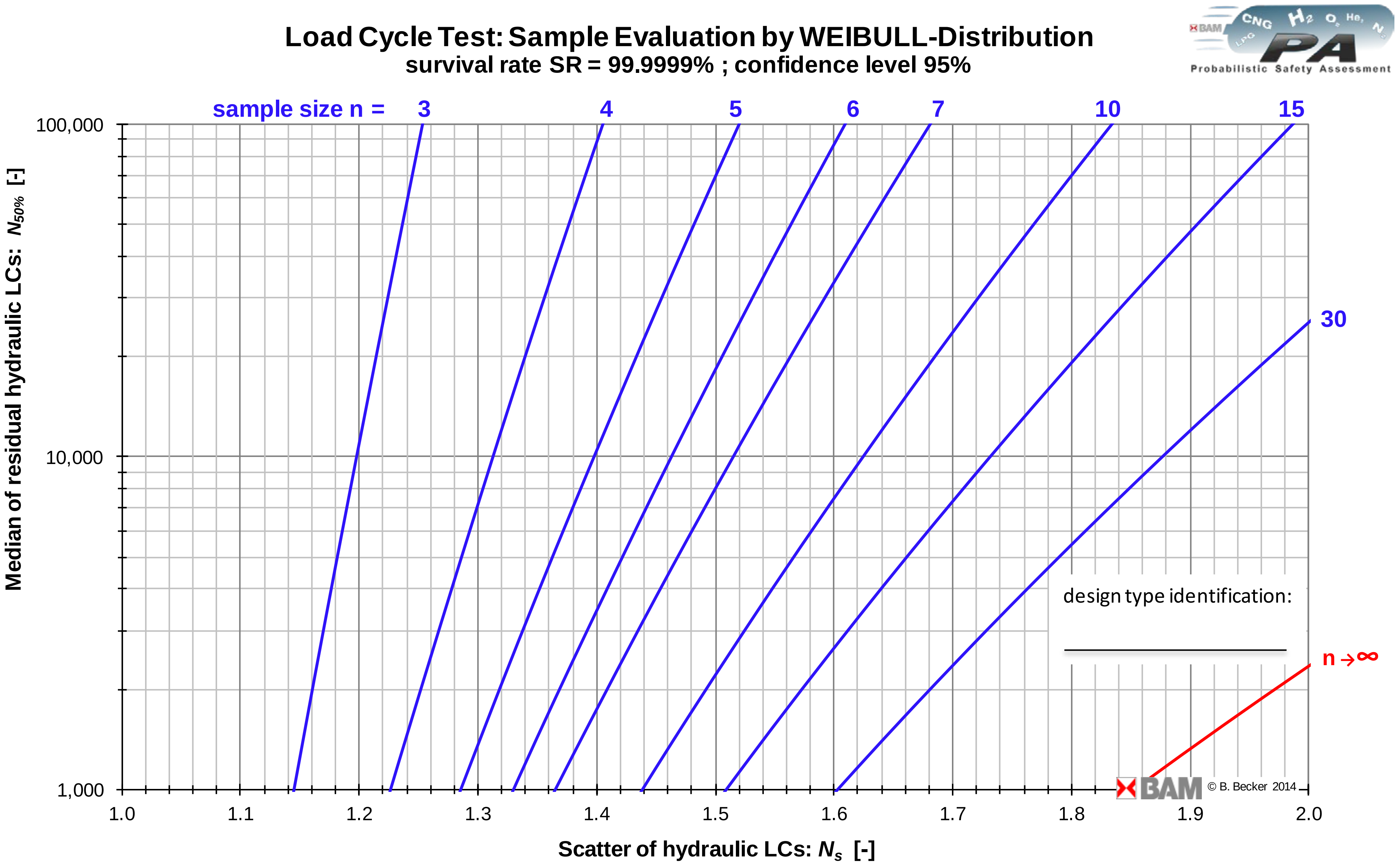

Fig. SAS-4: Performance chart for a graphical assessment of results from sample testing by LCT, survival rate SR = 99.9999%